Stony Point High School
IB Diploma Programme
Course Syllabus

IB Math Applications and Interpretation SL

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I. Course Description:
This course recognizes the increasing role that mathematics and technology play in a diverse range of fields in a data-rich world. As such, it emphasizes the meaning of mathematics in context by focusing on topics that are often used as applications or in mathematical modelling. To give this understanding a firm base, this course also includes topics that are traditionally part of a pre-university mathematics course such as calculus and statistics.

The course makes extensive use of technology to allow students to explore and construct mathematical models. Mathematics: applications and interpretation will develop mathematical thinking, often in the context of a practical problem and using technology to justify conjectures. Students who choose Mathematics: applications and interpretation at SL should enjoy seeing mathematics used in real-world contexts and to solve real-world problems. (Mathematics Applications and Interpretations Course Guide, IBO. 2019)

II. Prior Learning for Course:
It is expected that most students embarking on a DP mathematics course will have studied mathematics for at least 10 years. There will be a great variety of topics studied, and differing approaches to teaching and learning. Thus, students will have a wide variety of skills and knowledge when they start their mathematics course. Most will have some background in arithmetic, algebra, geometry, trigonometry, probability and statistics. (Mathematics Applications and Interpretations Course Guide, IBO. 2019)

III. Course Aims & Objectives:
The aims of all DP mathematics courses are to enable students to:
1. Develop a curiosity and enjoyment of mathematics, and appreciate its elegance and power 2. Develop an understanding of the concepts, principles and nature of mathematics
3. Communicate mathematics clearly, concisely and confidently in a variety of contexts
4. Develop logical and creative thinking, and patience and persistence in problem solving to instill confidence in using mathematics
5. Employ and refine their powers of abstraction and generalization
6. Take action to apply and transfer skills to alternative situations, to other areas of knowledge and to future developments in their local and global communities

7. Appreciate how developments in technology and mathematics influence each other
8. Appreciate the moral, social and ethical questions arising from the work of mathematicians and the applications of mathematics
9. Appreciate the universality of mathematics and its multicultural, international and historical perspectives
10. Appreciate the contribution of mathematics to other disciplines, and as a particular "area of knowledge" in the TOK course
11. Develop the ability to reflect critically upon their own work and the work of others 12. Independently and collaboratively extend their understanding of mathematics. (Mathematics Applications and Interpretations Course Guide, IBO. 2019)

IV. How the course will address TOK:

The relationship between each subject and theory of knowledge (TOK) is important and fundamental to the DP. The theory of knowledge course provides an opportunity for students to reflect on questions about how knowledge is produced and shared, both in mathematics and also across different areas of knowledge. It encourages students to reflect on their assumptions and biases, helping them to become more aware of their own perspective and the perspectives of others and to become "inquiring, knowledgeable and caring young people" (IB mission statement).

As part of their theory of knowledge course, students are encouraged to explore tensions relating to knowledge in mathematics. As an area of knowledge, mathematics seems to supply a certainty perhaps impossible in other disciplines and in many instances provides us with tools to debate these certainties. This may be related to the "purity" of the subject, something that can sometimes make it seem divorced from reality. Yet mathematics has also provided important knowledge about the world and the use of mathematics in science and technology has been one of the driving forces for scientific advances.

Despite all its undoubted power for understanding and change, mathematics is in the end a puzzling phenomenon. A fundamental question for all knowers is whether mathematical knowledge really exists independently of our thinking about it. Is it there, "waiting to be discovered", or is it a human creation? Indeed, the philosophy of mathematics is an area of study in its own right.

The following is one sample TOK question from each of the five topics we will address in this course. For a comprehensive list, refer to the IB Subject Guide for this class.

Topic 1 TOK Question: Do the names that we give things impact how we understand them? For instance, what is the impact of the fact that some large numbers are named, such as the googol and the googolplex, while others are represented in this form?

Topic 2 TOK Question: Descartes showed that geometric problems could be solved algebraically

and vice versa. What does this tell us about mathematical representation and mathematical knowledge?

Topic 3 TOK Question: What is an axiomatic system? Are axioms self evident to everybody?

Topic 4 TOK Question: Why have mathematics and statistics sometimes been treated as separate subjects? How easy is it to be misled by statistics? Is it ever justifiable to purposely use statistics to mislead others?

Topic 5 TOK Question: What value does the knowledge of limits have? Is infinitesimal behaviour applicable to real life? Is intuition a valid way of knowing in mathematics?

V. How the course will address CAS:

CAS and mathematics can complement each other in a number of ways. Mathematical knowledge provides an important key to understanding the world in which we live, and the mathematical skills and techniques students learn in the mathematics courses will allow them to evaluate the world around them which will help them to develop, plan and deliver CAS experiences or projects.

An important aspect of the mathematics courses is that students develop the ability to systematically analyze situations and can recognize the impact that mathematics can have on the world around them. An awareness of how mathematics can be used to represent the truth enables students to reflect critically on the information that societies are given or generate, and how this influences the allocation of resources or the choices that people make. This systematic analysis and critical reflection when problem solving may be inspiring springboards for CAS projects.

Students may also draw on their CAS experiences to enrich their involvement in mathematics both within and outside the classroom, and mathematics teachers can assist students in making links between their subjects and students' CAS experiences where appropriate. Purposeful discussion about real CAS experiences and projects will help students to make these links. The challenge and enjoyment of CAS can often have a profound effect on mathematics students, who might choose, for example, to engage with CAS in the following ways:

- plan, write and implement a "mathematics scavenger hunt" where younger students tour the school answering interesting mathematics questions as part of their introduction to a new school
- as a CAS project students could plan and carry out a survey, create a database and analyze the results, and make suggestions to resolve a problem in the students' local area. This might be, for example, surveying the availability of fresh fruit and vegetables within a community, preparing an action plan with suggestions of how to increase availability or access, and presenting this to a local charity or community group

• taking an element of world culture that interests students and designing a miniature Earth (if the world were 100 people) to express the trend(s) numerically.

It is important to note that a CAS experience can be a single event or may be an extended series of events. (Mathematics Applications and Interpretations Course Guide, IBO. 2019)

VI. How the course will address Approaches to Learning skills:
A major emphasis of the IB Programme is implementation of the following approaches to learning: thinking skills, communication skills, social skills, self-management skills, and research skills. To be successful in life after high school, you'll need to master the whole set. You will have the opportunity to practice these skills during your time in this class. Here's how:

Thinking skills: This one is easy. Math is logic. In fact it is the very basis for much of the logical thinking you will need to do for the rest of your life. This is a skill that can only be mastered by repetition and discipline. Do your homework, but do not do it begrudgingly. Find joy and peace in your ability to successfully traverse a complex problem with numerous steps. Not many people can. There is zen to be found as a result of hard work. We will find it.

Communication skills and social skills: Nobody can do really difficult mathematics alone or without some assistance along the way. Form a study group to work problems together and to bounce ideas. Come to me during tutorials and tell me how I can help you. If you are having trouble, tell someone. We are all in this together. Take this with you for the rest of your life no matter what it is you are doing: NEVER FAIL ALONE.

Self-management skills: You will have notes upon notes upon notes for this class. You will have solved problems, tests, knowledge checks, and sample IB external assessment problems. You will have so many papers that if you don't have a system to manage them, you will drown in them. Get a binder for this course and organize it. Every time a paper is handed to you, it is for a reason. Work out the organizational system now, and you will thank yourself in May when we are preparing for the external assessment.

Research skills: This class requires an internal assessment which makes up 20% of your total grade. Most of the projects will use statistical methods to analyze some set of data. The math and the writing are the easy parts. Obtaining a valid data set is the difficult part. You've likely never done this sort of research before, but it may be the most valuable thing you learn to do in this class. We will tackle this process together, and I promise you will have an end product that you will be proud of for years to come.

VII. The approach to teaching the course:
This is my eleventh year of teaching, and I have a class structure which I've found maximizes the probability of student success. That is not to say that I am not flexible. If there is something I am doing which doesn't work, we will change it. My only priority in this course is to ensure that you receive the maximum number of points possible to go toward your IB Diploma requirements.

Any other concern is secondary.

That said, a typical class sequence will be as follows: you will view videos for the lesson before class, during class we will work more examples, answer questions, and do activities, and concluding with a set of homework problems (and usually some time for you to work on said problems).

At the end of each unit, we will spend a day reviewing the content from the unit. The next class we will take an assessment which will contain a combination of questions from previous IB exams and problems from your homework.

Similarly, from time to time, we will engage in TOK related discussions and work on your internal assessment. Additionally, a number of days have been reserved for you to work on your internal assessment exclusively.

VIII. How the course will address the Learner Profile:
"The aim of all IB programmes is to develop internationally minded people who, recognizing their common humanity and shared guardianship of the planet, help to create a better and more peaceful world."

You will be expected to exemplify all ten of the learner profile attributes in this class. In particular, you will become a better inquirer as you develop the basis for your internal assessment and question the fundamental nature of mathematics. You will become more knowledgeable as you address an entire spectrum of mathematics ranging from concepts you first learned in the eighth grade and concluding with differential calculus. You will become a better thinker as you learn to analyze a mathematical problem to determine which tool is the most appropriate and how it should be implemented. You will become a better risk-taker as you embrace failure as the only means of success in mathematics. Finally, you will become more reflective as you analyze your own strengths and weaknesses in mathematics. It is my hope that through these things you will become a better human being.

IX. Assessment details for Internal and External Requirements:
The following summarizes how IB will determine your grade for Math Applications and Interpretations SL.

Assessment component	Weighting
External assessment (3 hours) **Paper 1 (90 minutes)** Technology required. (80 marks) Compulsory short-response questions based on the syllabus. (80 marks)	**80%** **40%**
Paper 2 (90 minutes) Technology required. (80 marks) Compulsory extended-response questions based on the syllabus. (80 marks)	**40%**
Internal assessment This component is internally assessed by the teacher and externally moderated by the IB at the end of the course. **Mathematical exploration** Internal assessment in mathematics is an individual exploration. This is a piece of written work that involves investigating an area of mathematics. (20 marks)	**20%**

X. Grading Policy & Scale:

Your grade in this class will come from two categories: Minor Grades (homework) and Major Grades (Quizzes and Tests). Major Grades are 40% of your total grade and the remaining 60% will come from daily participation/homework. Tests will consist of Paper 1 and Paper 2 type questions to prepare you for examinations for this class.

XI. Course Sequence:

During the two years of the course, we will address five topics and we will have one exploration project.

Syllabus component	Suggested teaching hours—SL
Topic 1—Number and algebra	16
Topic 2—Functions	31
Topic 3—Geometry and trigonometry	18
Topic 4—Statistics and probability	36
Topic 5—Calculus	19
The "toolkit" and Mathematical exploration Investigative, problem-solving and modelling skills development leading to an individual exploration. The exploration is a piece of written work that involves investigating an area of mathematics.	30
Total teaching hours	150

XII. IA Checkpoint dates and final IA Deadlines:

The internally-assessed component in this course is a mathematical exploration. This is a short report written by the student based on a topic chosen by him or her, and it should focus on the mathematics of that particular area. The emphasis is on mathematical communication

(including formulae, diagrams, graphs, tables and so on), with his or her own focus, with the teacher providing feedback via, for example, discussion and interview. This will allow the students to develop areas of interest to them without a time constraint as in an examination, and allow all students to experience a feeling of success.

The final report should be approximately 12-20 pages long with double line spacing. It can be either word processed or handwritten. Students should be able to explain all stages of their work in such a way that demonstrates clear understanding. While there is no requirement that students present their work in class, it should be written in such a way that their peers would be able to follow it fairly easily. The report should include a detailed bibliography, and sources need to be referenced in line with the IB academic honesty policy. Direct quotes must be acknowledged.

The aims of the Mathematics: applications and interpretation course at both SL and HL are carried through into the objectives that are formally assessed as part of the course, through either written examination papers or the exploration, or both. In addition to testing the objectives of the course, the exploration is intended to provide students with opportunities to increase their understanding of mathematical concepts and processes, and to develop a wider appreciation of mathematics. These are noted in the aims of the course. It is intended that, by doing the exploration, students benefit from the mathematical activities undertaken and find them both stimulating and rewarding. It will enable students to acquire the attributes of the IB learner profile. The specific purposes of the exploration are to:

- develop students' personal insight into the nature of mathematics and to develop their ability to ask their own questions about mathematics
- provide opportunities for students to complete a piece of mathematical work over an extended period of time
- enable students to experience the satisfaction of applying mathematical processes independently
- provide students with the opportunity to experience for themselves the beauty, power and usefulness of mathematics
- encourage students, where appropriate, to discover, use and appreciate the power of technology as a mathematical tool
- enable students to develop the qualities of patience and persistence, and to reflect on the significance of their work
- provide opportunities for students to show, with confidence, how they have developed mathematically.

Year 1 students will begin the IA at the end of the school year. Year 2 students will turn in the final draft before spring break. There will be time in class to work on the IA in both years.

XIII. Sample exam questions

The following are sample questions from Topic 2: Functions

Paper 1

Professor Vinculum investigated the migration season of the Bulbul bird from their natural wetlands to a warmer climate.

He found that during the migration season their population, P could be modelled by $P = 1350 + 400(1.25)^{-t}$, $t \geq 0$, where t is the number of days since the start of the migration season.

(a) Find the population of the Bulbul birds,

(i) at the start of the migration season.

(ii) in the wetlands after 5 days. [3]

(b) Calculate the time taken for the population to decrease below 1400. [2]

(c) According to this model, find the smallest possible population of Bulbul birds during the migration season. [1]

Paper 2

The braking distance of a vehicle is defined as the distance travelled from where the brakes are applied to the point where the vehicle comes to a complete stop.

The speed, $s\ \text{m s}^{-1}$, and braking distance, $d\ \text{m}$, of a truck were recorded. This information is summarized in the following table.

Speed, $s\ \text{m s}^{-1}$	0	6	10
Braking distance, $d\ \text{m}$	0	12	60

This information was used to create Model A, where d is a function of s, $s \geq 0$.

Model A: $d(s) = ps^2 + qs$, where $p, q \in \mathbb{Z}$

At a speed of $6\ \text{m s}^{-1}$, Model A can be represented by the equation $6p + q = 2$.

(a) (i) Write down a second equation to represent Model A, when the speed is $10\ \text{m s}^{-1}$.

(ii) Find the values of p and q [4]

(b) Find the coordinates of the vertex of the graph of $y = d(s)$. [2]

(c) Using the values in the table and your answer to part (b), sketch the graph of $y = d(s)$ for $0 \leq s \leq 10$ and $-10 \leq d \leq 60$, clearly showing the vertex. [3]

(d) Hence, identify why Model A may not be appropriate at lower speeds. [1]

Additional data was used to create Model B, **a revised model** for the braking distance of a truck.

Model B: $d(s) = 0.95s^2 - 3.92s$

(e) Use Model B to calculate an estimate for the braking distance at a speed of $20\ \text{m s}^{-1}$. [2]

The actual braking distance at $20\ \text{m s}^{-1}$ is $320\ \text{m}$.

(f) Calculate the percentage error in the estimate in part (e). [2]

XIV. Stony Point Academic Honesty Policy:

All students are expected to abide by the SPHS Academic Honesty Policy. This policy is available on the SPHS IB website at www.stonypointib.com under Handbooks.

Collaboration will be our biggest asset in this class. Together we will ensure that every one of us does well on both the internal and external assessments. I expect you will work together on all homework assignments, studying for class and external assessments, and editing the internal assessment. I will consider any collaboration on knowledge checks and tests to be cheating, and consequences will be determined by the SPHS Academic Honesty Policy.

XV. Writing an Extended Essay in Mathematics Applications and Interpretation SL: Writing an EE in mathematics can be challenging, but I'm happy to help you if this is something you are interested in doing. Please talk to me individually if you're interested in writing your EE about math.

XVI. Classroom Supplies, Policy, & Procedures:

Supplies: Pencil, eraser, notebook paper, binder/folder, and calculator for use at home (TI-84 recommended). The use of a graphing calculator (GDC) is integral for success in this course. You will always have access to one in my classroom, but I cannot check one out to you.

Make Up Work Policy: Students should access the webpage for the class calendar to find missed lessons and assignments. It is the student's responsibility to find out what he or she missed from me. Students may also come by my room the day they return to school from an absence in order to get their make-up work, even if they DO NOT have class that day. Students will have the exact number of days he or she missed to make up the assignment. All makeup work must be labeled with the word ABSENT, the date of the absence and any page numbers for the work. If you were absent on the review day, you should get the review from the webpage or during tutorials, so you are prepared for the test on test day. If a student is absent for school sponsored absences (such as sports, field trips, etc), students are REQUIRED to get their work in advance.

Re-Take Policy forTest Grades: Must show that review has been completed and no outstanding work to be completed. Retake within 1 week up to 70.

APPENDIX A: Prerequisite Skills

Number systems: natural numbers $\mathbb{N}$; integers, $\mathbb{Z}$; rationals, $\mathbb{Q}$, and irrationals; real numbers, $\mathbb{R}$
SI (Système International) units for mass, time, length, area and volume and their derived units, eg. speed
Rounding, decimal approximations and significant figures, including appreciation of errors Definition and elementary treatment of absolute value (modulus), a
Use of addition, subtraction, multiplication and division using integers, decimals and fractions, including order of operations
Prime numbers, factors (divisors) and multiples
Greatest common factor (divisor) and least common multiples (HL only)
Simple applications of ratio, percentage and proportion
Manipulation of algebraic expressions, including factorization and expansion Rearranging formulae
Calculating the numerical value of expressions by substitution
Evaluating exponential expressions with simple positive exponents
Evaluating exponential expressions with rational exponents (HL only)
Use of inequalities, $<,\leq,>,\geq$, intervals on the real number line
Simplification of simple expressions involving roots (surds or radicals)
Expression of numbers in the form $a\times10k$, $1\leq a<10$, $k\in\mathbb{Z}$
Familiarity with commonly accepted world currencies
Solution of linear equations and inequalities
Solution of quadratic equations and inequalities with rational coefficients (HL only) Solving systems of linear equations in two variables
Concept and basic notation of sets. Operations on sets: union and intersection Addition and subtraction of algebraic fractions (HL only).
Graphing linear and quadratic functions using technology
Mappings of the elements of one set to another. Illustration by means of sets of ordered pairs, tables, diagrams and graphs.
Pythagoras' theorem and its converse
Mid-point of a line segment and the distance between two points in the Cartesian plane Geometric concepts: point, line, plane, angle
Angle measurement in degrees, compass directions
The triangle sum theorem
Right-angle trigonometry, including simple applications for solving triangles Three-figure bearings
Simple geometric transformations: translation, reflection, rotation, enlargement The circle, its centre and radius, area and circumference. The terms diameter, arc, sector, chord, tangent and segment
Perimeter and area of plane figures. Properties of triangles and quadrilaterals, including parallelograms, rhombuses, rectangles, squares, kites and trapezoids; compound shapes
Familiarity with three-dimensional shapes (prisms, pyramids, spheres, cylinders and cones)

Volumes and surface areas of cuboids, prisms, cylinders, and compound three-dimensional shapes

The collection of data and its representation in bar charts, pie charts, pictograms, and line graphs

Obtaining simple statistics from discrete data, including mean, median, mode, range Calculating probabilities of simple events

Venn diagrams for sorting data

Tree diagrams

Speed = distance/time